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Hexagonal Difference Prime Labeling of Some Snake Graphs

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Abstract- The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. Here the vertices of the graph are labeled with hexagonal numbers and the edges are labeled with absolute difference of the end vertex labels. Here the greatest common incidence number of a vertex of degree greater than one is defined as the g c d of the labels of the incident edges. If the greatest common incidence number of each vertex of degree greater than one is 1, then the graph admits hexagonal difference prime labeling. Here we characterize some snake graphs for hexagonal difference prime labeling.

Index Terms- Graph labeling; hexagonal numbers; greatest common incidence number ; snake graph..

1. INTRODUCTION

In this paper we deal with graphs that are connected, simple, finite and undirected. The symbol V and E denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated the hexagonal difference prime labeling of some snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (g c i n) of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 n^{th} hexagonal number is n(2n-1), where n is a positive integer. The hexagonal numbers are 1, 6, 15, 28, 45, 66--------------.

2. MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

f : V(G) → {1,6,15,28 ,------------------,p(2p-1)} by $f(v_i) = i(2i-1)$, for every i from 1 to p and define a 1-1 mapping f_{hdpl}^* : E(G) → set of natural numbers N by $f_{hdpl}^*(uv)$ = | f(u)-f(v) |. The induced function f_{hdpl}^* is said to be hexagonal difference prime labeling, if the g c i n of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits hexagonal difference prime labeling is called hexagonal difference prime graph.

Theorem: 2.1 Triangular snake T_n, admits hexagonal difference prime labeling.

Proof : Let G = T_n and let v_1,v_2,---------------,v_{2n-1} are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-3.

Define a function

f : V → {1,6,15,28 ,------------------,(2n-1)(4n-3)} by

$f(v_i) = i(2i-1)$, i = 1,2,------,2n-1.

For the vertex labeling f, the induced edge labeling f_{hdpl}^* is defined as follows

$f_{hdpl}^*(v_i\, v_{i+1})$ = (4i+1), i = 1,2,----,2n-2.

$f_{hdpl}^*(v_{2i-1}\, v_{2i+1})$ = (16i-2),i =1,2,------,n-1.

Clearly f_{hdpl}^* is an injection.

g c i n of (v_1) = g c d of { $f_{hdpl}^*(v_1\, v_2)$, $f_{hdpl}^*(v_1\, v_3)$ }
= g c d of { 5, 14}= 1.

g c i n of (v_{i+1}) = g c d of { $f_{hdpl}^*(v_i\, v_{i+1})$, $f_{hdpl}^*(v_{i+1}\, v_{i+2})$ }
= g c d of { (4i+1) , (4i+5) }
=1, i = 1,2,------------,2n-3.

g c i n of (v_{2n-1})= g c d of { $f_{hdpl}^*(v_{2n-2}\, v_{2n-1})$, $f_{hdpl}^*(v_{2n-3}\, v_{2n-1})$ }
= g c d of {8n-7, 16n-18 }
= g c d of {8n-11, 8n-7 }= 1.

So, g c i n of each vertex of degree greater than one is 1.

Hence T_n , admits hexagonal difference prime labeling.

Theorem: 2.2 Quadrilateral snake Q_n, admits hexagonal difference prime labeling.

Proof : Let G = Q_n and let v_1, v_2,---------------,v_{3n-2} are the vertices of G.

Here |V(G)| = 3n-2 and |E(G)| = 4n-4.

Define a function

f : V → {1,6,15,28 ,------------------,(3n-2)(6n-5)} by

$f(v_i)$ = i(2i-1) , i = 1,2,------,3n-2.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,3n-3.

$f^*_{hdpl}(v_{3i-2}\, v_{3i+1})$= (36i-9),i = 1,2,---------,n-1.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) = g c d of { $f^*_{hdpl}(v_1\, v_2)$, $f^*_{hdpl}(v_1\, v_4)$ }

= g c d of { 5, 27}= 1.

g c i n of (v_{i+1}) = 1, i = 1,2,--------------,3n-4.

g c i n of (v_{3n-2})= g c d of { $f^*_{hdpl}(v_{3n-3}\, v_{3n-2})$, $f^*_{hdpl}(v_{3n-5}\, v_{3n-2})$ }

= g c d of {12n-11, 36n-45 }

= g c d of {12n-11, 12n-23 }= 1.

So, g c i n of each vertex of degree greater than one is 1.

Hence T_n , admits hexagonal difference prime labeling.

Theorem: 2.3 Double triangular snake D(T_n), admits hexagonal difference prime labeling.

Proof : Let G = D(T_n), and let v_1, v_2,----------,v_{3n-2} are the vertices of G.

Here |V(G)| = 3n-2 and |E(G)| = 5n-5.

Define a function

f : V → {1,6,15,28 ,------------------,(3n-2)(6n-5)} by

$f(v_i)$ = i(2i-1) , i = 1,2,------,3n-2.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_{3i-2}\, v_{3i-1})$= (12i-7), i = 1,2,------------,n-1.

$f^*_{hdpl}(v_{3i-1}\, v_{3i+1})$= (24i-2), i = 1,2,------------,n-1.

$f^*_{hdpl}(v_{3i-2}\, v_{3i})$= (24i-10), i = 1,2,------------,n-1.

$f^*_{hdpl}(v_{3i}\, v_{3i+1})$= (12i+1), i = 1,2,------------,n-1.

$f^*_{hdpl}(v_{3i-2}\, v_{3i+1})$= (36i-9), i = 1,2,------------,n-1.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) = g c d of { $f^*_{hdpl}(v_1\, v_2)$, $f^*_{hdpl}(v_1\, v_3)$ }

= g c d of { 5, 14}= 1.

g c i n of (v_{3i-1}) = g c d of { $f^*_{hdpl}(v_{3i-2}\, v_{3i-1})$, $f^*_{hdpl}(v_{3i-1}\, v_{3i+1})$ }

= g c d of { (12i-7) , (24i-2) }

= g c d of { (12i-7) , (12) }

= g c d of {5,12} = 1, i = 1,2,----------,n-1.

g c i n of (v_{3i}) = g c d of { $f^*_{hdpl}(v_{3i-2}\, v_{3i})$, $f^*_{hdpl}(v_{3i}\, v_{3i+1})$ }

= g c d of { (12i+1) , (24i-10) }

= g c d of { (12i-11) , (12i+1) }

= g c d of {12,12i-11}

= g c d of {1,12} = 1, i = 1,2,----------,n-1.

g c i n of (v_{3i+1}) = g c d of { $f^*_{hdpl}(v_{3i+1}\, v_{3i})$, $f^*_{hdpl}(v_{3i+2}\, v_{3i+1})$ }

= g c d of { (12i+1) , (12i+5) }

= g c d of { 4 , (12i+1) }

= 1, i = 1,2,----------------,n-2.

g c i n of (v_{3n-2})= g c d of { $f^*_{hdpl}(v_{3n-2}\, v_{3n-3})$, $f^*_{hdpl}(v_{3n-2}\, v_{3n-4})$ }

= g c d of {12n-11, 24n-26 }

= g c d of {12n-15, 12n-11 }

= g c d of {4, 12n-15}

= g c d of {1,4} =1.

So, g c i n of each vertex of degree greater than one is 1.

Hence D(T_n), admits hexagonal difference prime labeling.

Theorem: 2.4 Alternate triangular snake A(T_n), admits hexagonal difference prime labeling.

Proof : Let G = A(T_n)

Here |V(G)| = p and |E(G)| = q.

Define a function f : V → {1,6,15,28 ,------------------,(p)(2p-1)} by

$f(v_i)$ = i(2i-1) , i = 1,2,------,p.

Case(i) n is even and triangle starts from the first vertex.

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\, v_{i+1})$ = (4i+1), i = 1,2,-----------,($\frac{3n}{2}$ – 1).

$f^*_{hdpl}(v_{3i-2}\, v_{3i})$= (24i-10), i = 1,2,------------,$\frac{n}{2}$.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) = g c d of { $f^*_{hdpl}(v_1\, v_2)$, $f^*_{hdpl}(v_1\, v_3)$ }

= g c d of { 5, 14}= 1.

g c i n of (v_{i+1}) =1, i = 1,2,--------------,$\frac{3n-4}{2}$.

g c i n of ($v_{3n/2}$)= g c d of { $f^*_{hdpl}(v_{(\frac{3n-2}{2})}\, v_{(\frac{3n}{2})})$, $f^*_{hdpl}(v_{(\frac{3n-4}{2})}\, v_{(\frac{3n}{2})})$ }

= g c d of {6n-3, 12n-10 }

= g c d of {6n-7, 6n-3 }

= g c d of {6n-7, 4 }

= g c d of {1,4 } = 1.

So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits hexagonal difference prime labeling.

Case(ii) n is even and triangle starts from the second vertex.

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,$\frac{3n-4}{2}$.

$f^*_{hdpl}(v_{3i-1}\, v_{3i+1})$= (24i-2), i = 1,2,------------,$\frac{n-2}{2}$.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) =1, i = 1,2,-------------,$\frac{3n-6}{2}$.

So, g c i n of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits hexagonal difference prime labeling.

Case(iii) n is odd and triangle starts from the first vertex.

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n-2$.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,$\frac{3n-3}{2}$.

$f^*_{hdpl}(v_{3i-2}\, v_{3i})$= (24i-10), i = 1,2,------------,$\frac{n-1}{2}$.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_1) = g c d of { $f^*_{hdpl}(v_1\, v_2)$, $f^*_{hdpl}(v_1\, v_3)$ }

= g c d of { 5, 14}= 1.

g c i n of (v_{i+1}) =1, i = 1,2,-------------,$\frac{3n-5}{2}$.

So, g c i n of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits hexagonal difference prime labeling.

Case(iv) n is odd and triangle starts from the second vertex.

Here $|V(G)| = \frac{3n-1}{2}$ and $|E(G)| = 2n-2$.

For the vertex labeling f, the induced edge labeling f^*_{hdpl} is defined as follows

$f^*_{hdpl}(v_i\, v_{i+1})$ = (4i+1), i = 1,2,------------,$\frac{3n-3}{2}$.

$f^*_{hdpl}(v_{3i-1}\, v_{3i+1})$= (24i-2), i = 1,2,------------,$\frac{n-1}{2}$.

Clearly f^*_{hdpl} is an injection.

g c i n of (v_{i+1}) =1, i = 1,2,-------------,$\frac{3n-5}{2}$

g c i n of $(v_{(\frac{3n-1}{2})})$ = g c d of { $f^*_{hdpl}(v_{(\frac{3n-3}{2})}\, v_{(\frac{3n-1}{2})})$, $f^*_{hdpl}(v_{(\frac{3n-5}{2})}\, v_{(\frac{3n-1}{2})})$ }

= g c d of {6n-5, 12n-14 }

= g c d of {6n-9, 6n-5 }

= g c d of {6n-9, 4 } = 1.

So, g c i n of each vertex of degree greater than one is 1.

Hence $A(T_n)$, admits hexagonal difference prime labeling.

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